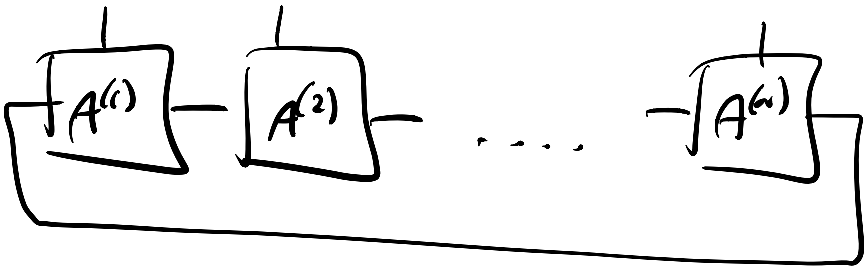
4. Periodic, translational invariant, & infinite MPS

MPS can also be defined with periodic boundary conditions (PBC):

Periodic MPS: A PBC MPS is of the form

$|\psi\rangle =$

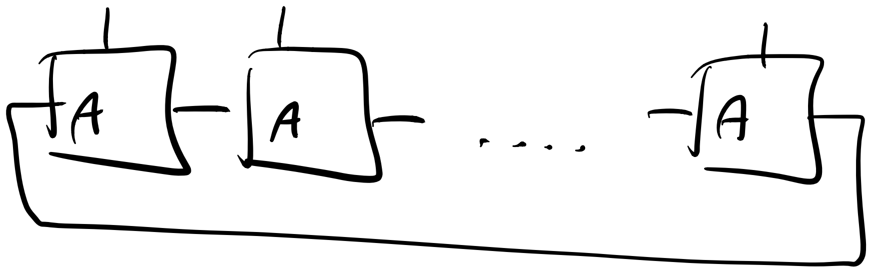


$$= \sum_{i_1,\dots,i_N} \mathrm{tr}\left[A^{i_1,(1)} \cdot A^{i_2,(2)} \cdot \ldots \cdot A^{i_N,(N)}\right] |i_1,\dots,i_N\rangle$$

In particular, PBC MPS can be chosen to be translational invariant:

Translational invariant (t.i.) PBC MPS:

A t.i. PBC MPS is obtained by choosing all tensors $A^{(n)}$ to be identical, $A^{(n)} \equiv A$:

$|\psi\rangle =$ 

$$= \sum_{i_1,\dots,i_N} \mathrm{tr}\left[A^{i_1} \cdot A^{i_2} \cdot \dots \cdot A^{i_N}\right] |i_1,\dots,i_N\rangle$$

We can also use this to define tr.inv. states in the thermodynamic limit $N \to \infty$, i.e., on infinite chains.

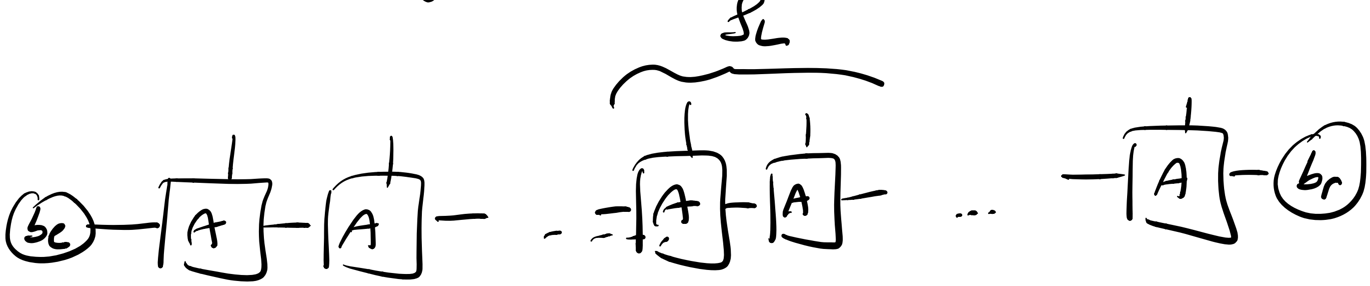
Will (probably) formalize this later.

<u>Intuition / Idea:</u>

(i) For infinite systems, we always only care about the reduced states (= exp. vals.) on some <u>finite</u> (but arb. big!) region.

(ii) For some region $(1\dots L)$, show that (under suitable conditions) ρ_L converges as $N \to \infty$.

(ii') Alternatively, use trw. OBC MPS:



and show that for $N \to \infty$, ρ_L is indep. of boundary cond. b_l, b_r.

(<u>Note</u>: This is stronger than (ii) – see later!)

<u>Important advantage</u> of trw. MPS:

State is described by $O(1)$ parameters, indep. of system size N, and we can describe state on <u>any</u> system size N with <u>one</u> set of parameters