V. Two dimensions:

Projected Entangled Pair States

1) Projected Entangled Pair States (PEPS)

MPS: - good approx. for ground states in 1D
- basis for powerful numerical methods
- framework for analytical modelling

Can we generalize this to 2D (and higher)?

What is the same?

What is different?

Basic intuition behind MPS:

Area law ⇒ entanglement distributed locally

Idea: Build 2D ansatz based on locality of entanglement.

a) Depicting

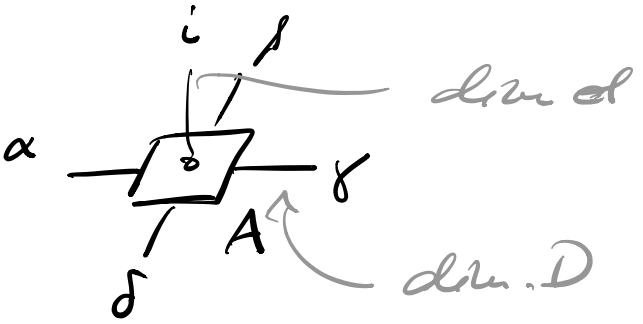
Consider 2D lattice

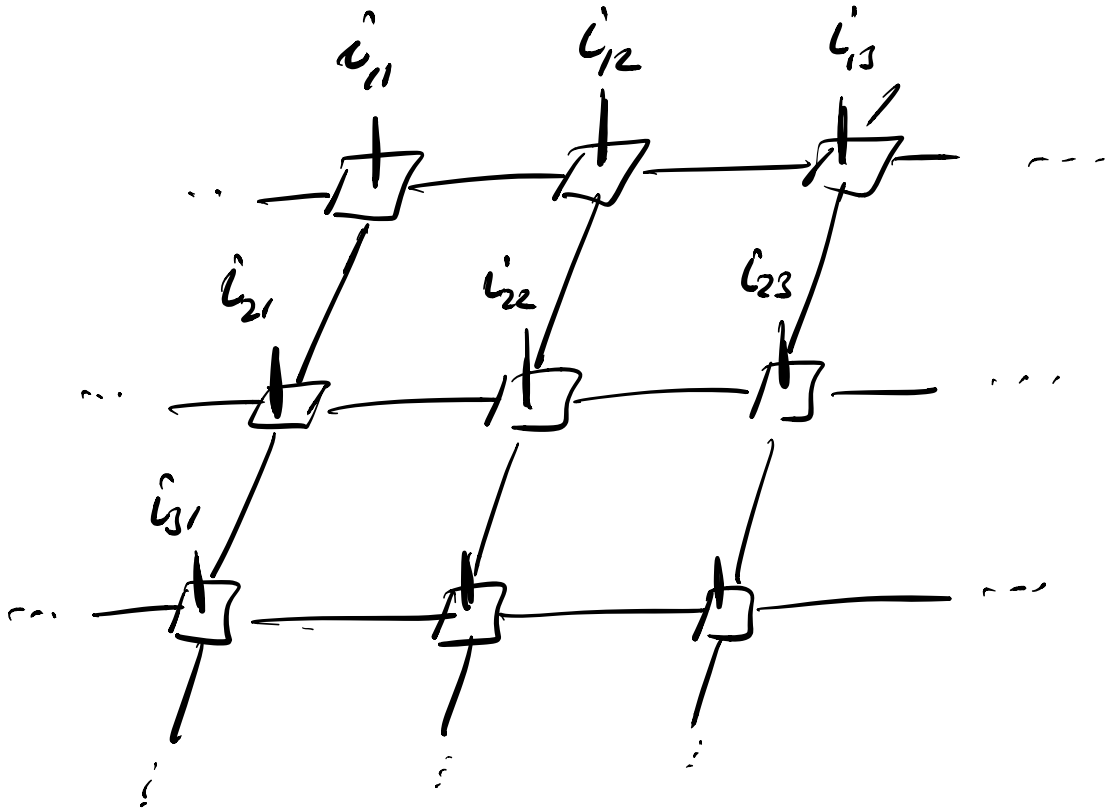
(here: square lattice: easiest to draw)

$$|\psi\rangle = \sum_{i_1,\dots,i_N} c_{i_1\dots i_N} |i_1,\dots,i_N\rangle$$

Express $c_{i_1\dots i_N}$ as 2D tensor network

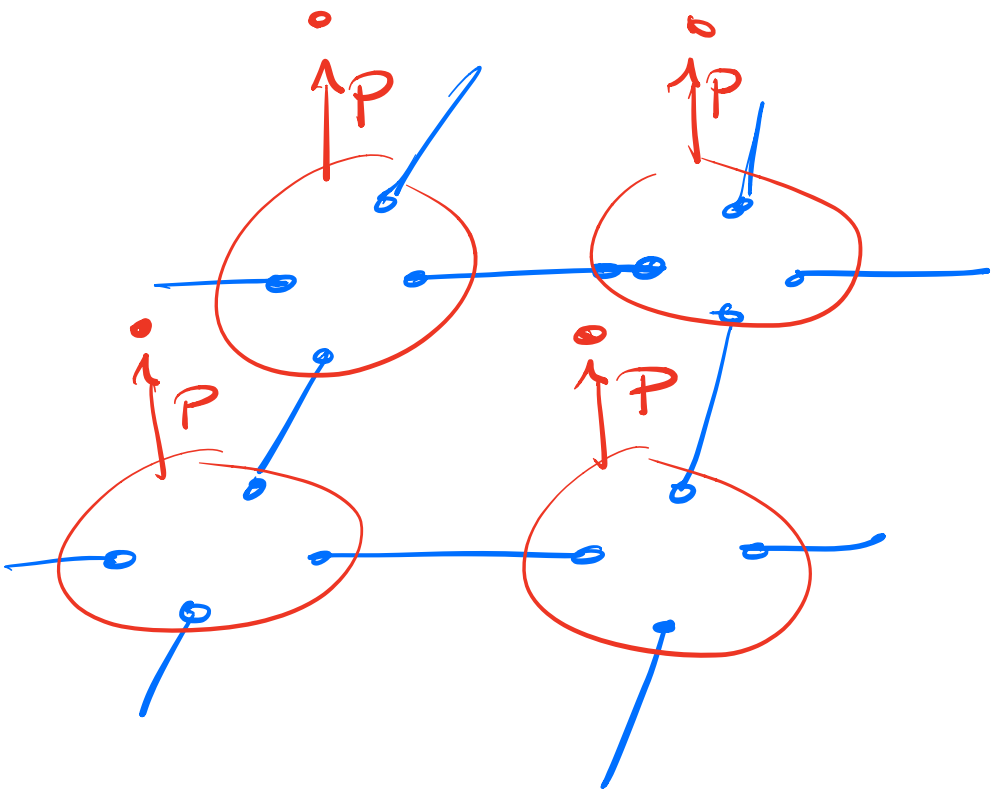
with tensor





As in 1D, we can either do this with PBC, terminate the boundaries, or choose site-dep. tensors $A^{[i,j]}$.

We can also express such states in the "AKLT way", i.e. with ent. states $|\omega\rangle$ and maps $P: \mathbb{C}^D \otimes \mathbb{C}^D \otimes \mathbb{C}^D \otimes \mathbb{C}^D \to \mathbb{C}^d$,



(e.g., $|\omega\rangle = \sum_{i=1}^{D} |i\rangle|i\rangle$; $P = \sum A^{i}_{\alpha\beta\gamma\delta} |i\rangle\langle\alpha,\beta,\gamma,\delta|$)

Because of this, these states are also called "Projected Entangled Pair States" (PEPS) — even though I need not be a projector — or simply "Tensor Network States" (TNS).

b) Approximability by PEPS

In 1D, area law ⇒ ent. distr. locally
⇒ good MPS approx.

In 2D, "area law ⇒ ent. local" does not automatically hold.

Do PEPS still provide a good approx. of ground states of 2D local Hamiltonians?

Yes! — Using more direct techniques (Trotter or Taylor expansion, which is truncated) we can get similar result:

Ground states are well approximated to error ε with a bond dimension

$$D \sim \left(\frac{N}{\varepsilon}\right)^{\log N}$$

i.e., <u>almost</u> polynomial.

(Terms & conditions apply, see arXiv: 1406.2973)

$\Rightarrow$ PEPS form a good class of states to approximate ground states.