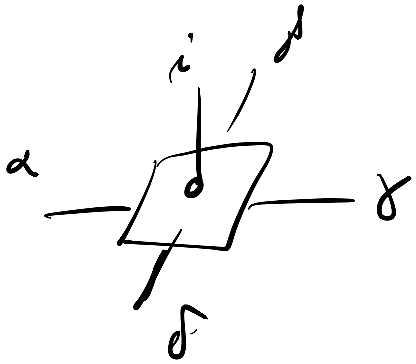
2. Examples

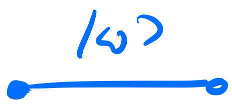
a) The GHZ state

$$|\psi\rangle = |0\dots0\rangle + |1\dots1\rangle$$

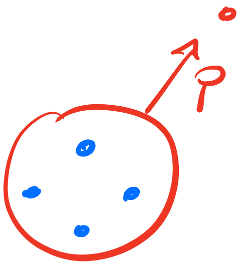
$$A^{i}_{\alpha\beta\gamma\delta} = \begin{cases} 1 : & i=\alpha=\beta=\gamma=\delta \\ 0 : & \text{otherwise} \end{cases}$$



b) AKLT state:



$$|\omega\rangle = \underbrace{\frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)}_{su(2)\ \text{singlet (spin } 1/2)}$$

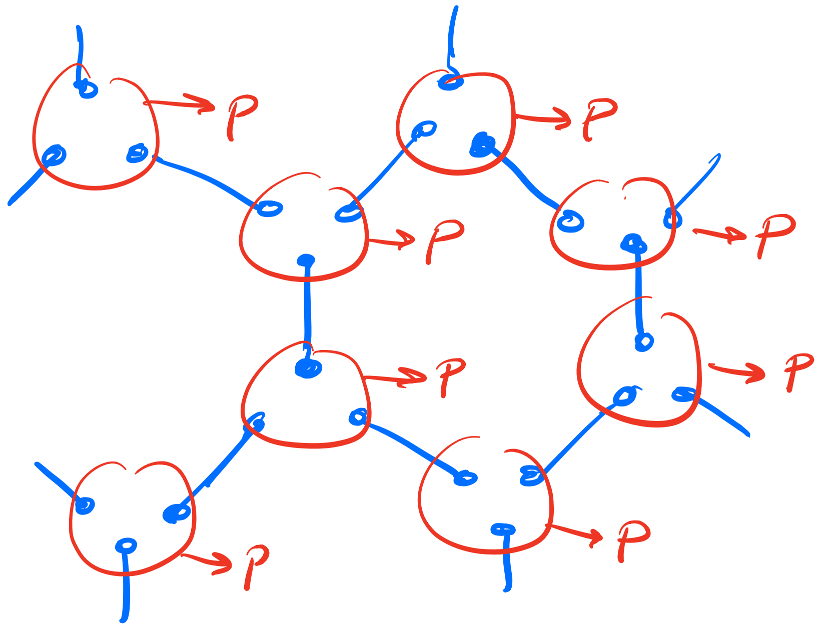


$\frac{1}{2}\otimes\frac{1}{2}\otimes\frac{1}{2}\otimes\frac{1}{2}$: spin 0, 1, 2

$$\underbrace{P = \Pi_{S=2}}_{\text{proj. onto perm.-sym. space!}}$$

$\longrightarrow$ su(2)-invariant spin-2 state.

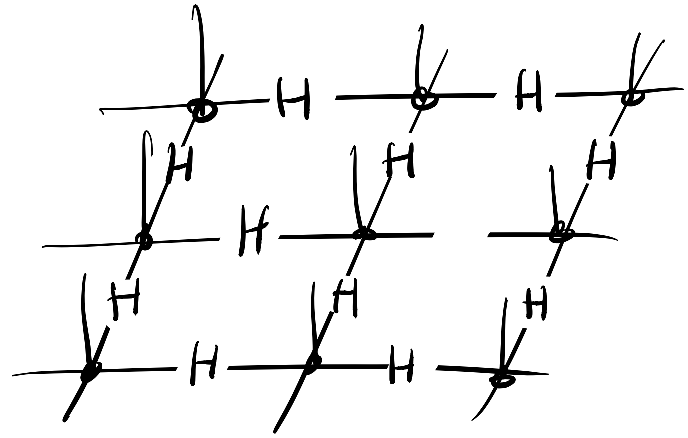
Alternatively: honeycomb lattice.

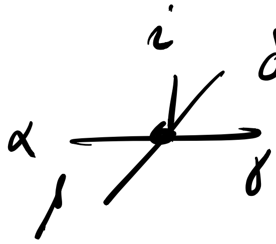


$$\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} = \frac{1}{2} \text{ or } \frac{3}{2} \Longrightarrow \underbrace{P = \Pi_{3/2}}_{\text{perm.-sym. space!}}$$

c) 2D cluster state

The 2D cluster state – obtained by acting with $CZ = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$ on $|+\rangle^{\otimes N}$ on all edges of the lattice is a PEPS with

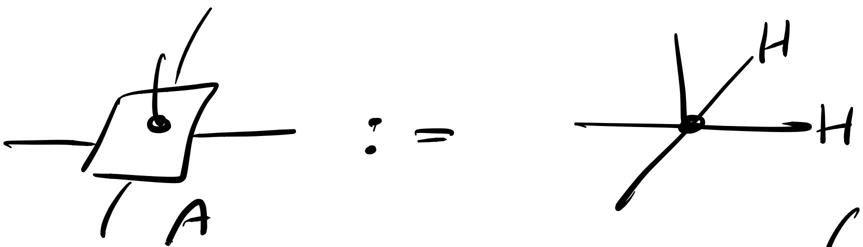



$= \delta_{i\alpha\beta\gamma\delta}$

$-H- \; = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

(Can be brought into canon. form

e.g. by blocking



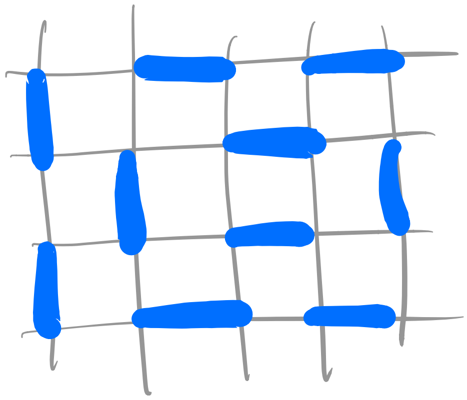
($\to$ Homework)

d) Topological models

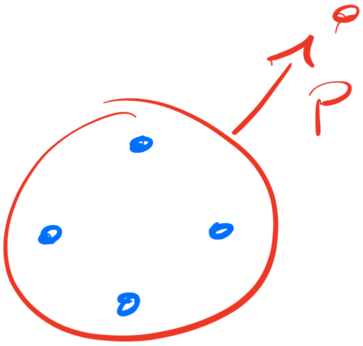
... hopefully later ...

e) The Resonating Valence Bond (RVB) state

<u>RVB state</u>: <u>equal weight</u> superpos. of all ways of covering lattice with NN singlets



$$|\omega\rangle = \underbrace{\frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)}_{\text{singlet}} + \underbrace{|22\rangle}_{\text{"no singlet"}}$$



$$P = |0\rangle\left[\langle 0222| + \langle 2022| + \dots\right] + |1\rangle\left[\langle 1222| + \langle 2122| + \dots\right]$$

(→ Homework)

f) PEPS from classical models

Let $H(s_1, \dots, s_N)$ be a classical statistical model, e.g. the 2D Ising model

$$H = -\sum_{\langle ij\rangle} s_i s_j \,, \quad s_i = \pm 1$$

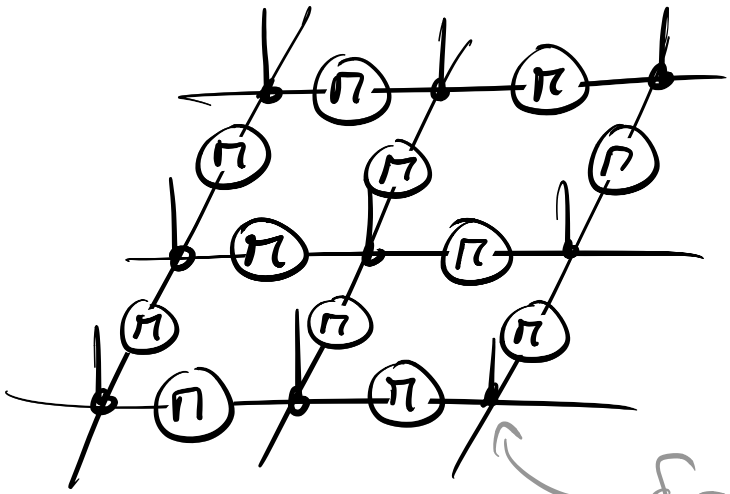
N.N.

Define

$$|\psi(\beta)\rangle = \sum_{s_1,\dots,s_N} e^{-\beta/2\, H(s_1,\dots,s_N)} |s_1,\dots,s_N\rangle$$

This state is a PEPS:

E.g. for the Ising model:

$|\psi(\beta)\rangle =$ 

δ-tensor

$$M = \begin{pmatrix} e^{\beta/2} & e^{-\beta/2} \\ e^{-\beta/2} & e^{\beta/2} \end{pmatrix}$$

(check: → HW)

(Can be written as "conventional" PEPS

with

or

,

i.e. $P = |0\rangle\langle\alpha,\alpha,\alpha,\alpha| + |1\rangle\langle\beta,\beta,\beta,\beta|$,

$|\alpha\rangle = \sqrt{\Pi}\,|0\rangle$; $|\beta\rangle = \sqrt{\Pi}\,|1\rangle$.)

For this state $|\psi(\beta)\rangle$,

$$\langle\psi(\beta)|\,\sigma_i^z\,\sigma_j^z\,|\psi(\beta)\rangle =$$

$$= \sum_{s_1,\dots,s_N} e^{-\beta H(s_1,\dots,s_N)} \langle s_1,\dots,s_N|\,\sigma_i^z\, s_j^z\,|s_1,\dots,s_N\rangle$$

$\Rightarrow$ $\langle \psi(\beta) | \sigma_i^z \sigma_j^z | \psi(\beta) \rangle$ equals the correlation in the classical Gibbs state

$e^{-\beta H}$!

(Same for more complex σ^z correlators.)

g) PEPS with critical correlations

2D classical models, e.g. Ising model, undergo phase transitions at some crit. temperature β_{crit}.

$\Rightarrow$ $e^{-\beta_{crit} H}$ has algebraically decaying correlations!

$$\langle \psi(\beta_{crit}) | \sigma_i^z \sigma_j^z | \psi(\beta_{crit}) \rangle \sim \frac{1}{|i-j|^{2\Delta_\sigma}}$$

with some exponent ("scaling dimension") Δ_σ.

⇒ PEPS can exhibit critical correlations.

Exponential clustering theorem: H gapped

⇒ ground state of H has exp. decaying correlations.

⇒ PEPS can describe states which are not ground states of gapped local Hamiltonians.