Lecture 260070 "Entanglement in quantum many-body systems" – SS 2021

— Exercise Sheet #3 —

Problem 5: Representations of translational invariant MPS

1. Let $|\psi\rangle$ be a translational invariant state on a chain of N sites, and let
$$|\psi\rangle = \sum A^{i_1,(1)} A^{i_2,(2)} \cdots A^{i_N,(N)} |i_1, i_2, \dots, i_N\rangle$$
be an open boundary condition (OBC) MPS representation of $|\psi\rangle$ with bond dimension D. Show that then, $|\psi\rangle$ can be also written as a translational invariant MPS with periodic boundary conditions,
$$|\psi\rangle = \sum \mathrm{tr}[B^{i_1} B^{i_2} \cdots B^{i_N}] |i_1, i_2, \dots, i_N\rangle \ ,$$
where the B^i are $ND \times ND$ matrices.
(*Hint:* This suggests that every B^i must contain the information from all $A^{i,(k)}$ – therefore, try to build B^i as a block matrix, where the blocks are the $A^{i,(k)}$ or zero. Note that $B^i B^j$ should contain $A^{i,(k)} A^{j,(k+1)}$ as blocks.)

2. What is the dimension of B^i in case the $A^{i_k,(k)}$ have different dimensions $D_{k-1} \times D_k$?

3. Can the result also be applied to turn a non-translational invariant *periodic* MPS representation of a translational invariant state $|\psi\rangle$ into a translational invariant MPS representation?

4. Consider the translational invariant state
$$|\psi\rangle = \frac{1}{\sqrt{2}} \big[|010101 \cdots 01\rangle + |101010 \cdots 10\rangle \big]$$
(on an even length chain). Find
   (a) an OBC MPS representation of $|\psi\rangle$,
   (b) a PBC MPS representation of $|\psi\rangle$,
   (c) a translational invariant PBC MPS representation of $|\psi\rangle$,

   ideally with the minimum possible bond dimension. (Bonus points for showing that the bond dimension is minimal.)

5. Try the same for the state
$$|\psi\rangle = \frac{1}{\sqrt{2}} \big[|010101 \cdots 01\rangle - |101010 \cdots 10\rangle \big] \ .$$
Which of the three representations (OBC, PBC, tinv PBC) does not exist, and why?

Note: Whenever we talk about finding an MPS representation of a state $|\psi\rangle$ in this problem, this is always up to proportionality.