Lecture 260070 "Entanglement in quantum many-body systems" – SS 2021

— Exercise Sheet #5 —

Problem 8: Transfer matrices and correlation length

Consider the following tinv. PBC MPS on an infinite chain:

- The GHZ state ($d = D = 2$):

$$A^0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad A^1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \ .$$

- The cluster state ($d = D = 2$):

$$A^0 = |0\rangle\langle +| \ , \quad A^1 = |1\rangle\langle -| \ .$$

- The AKLT state ($d = 3$, $D = 2$):

$$A^0 = \sigma_x \ , \quad A^1 = \sigma_y \ , \quad A^2 = \sigma_z \ .$$

1. For each of these MPS, compute the transfer matrix $\mathbb{E}$, its eigenvalues and eigenvectors, and the correlation length.
2. For the GHZ and the cluster state, compute the correlation functions $\langle \sigma_x^i \sigma_x^j \rangle$, $\langle \sigma_z^i \sigma_z^j \rangle$, and $\langle \sigma_x^i \sigma_z^j \rangle$ of the corresponding operators at positions i and j. (Here, $\langle M \rangle = \langle \psi | M | \psi \rangle / \langle \psi | \psi \rangle$.)
3. For the AKLT state on an infinite chain, compute the correlation function $\langle S_\alpha^i S_\beta^j \rangle$ as a function of the distance $i - j$ for any pair of operators S_α, $\alpha = x, y, z$ and S_β, $\beta = x, y, z$, where

$$S_x = \begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 \end{pmatrix} \ , \quad S_y = \begin{pmatrix} -1 & & \\ & 1 & \\ & & -1 \end{pmatrix} \ , \quad S_z = \begin{pmatrix} -1 & & \\ & -1 & \\ & & 1 \end{pmatrix} \ .$$

Problem 9: Hamiltonian as MPO

1. Show that any nearest neighbor Hamiltonian $H = \sum h_i$ (not necessarily translational invariant) can be written as an MPO. What is the minimum required bond dimension?
(*Hint:* Start by expressing $h_i = \sum_{\alpha=1}^{k_i} a_{i,\alpha} \otimes b_{i,\alpha}$. What is the minimal k_i, and how can this decomposition be found?)
2. Find an MPO for the Ising Hamiltonian with nearest and next-nearest neighbor interactions,

$$H = -J_1 \sum_i \sigma_z^i \sigma_z^{i+1} H = -J_2 \sum_i \sigma_z^i \sigma_z^{i+2} - h \sum_i \sigma_x^i \ .$$

3. Find an MPO for the Ising Hamiltonian with exponentially decaying interactions,

$$H = -\sum_{i<j} \lambda^{j-i} \sigma_z^i \sigma_z^j - h \sum_i \sigma_x^i \ .$$